where the shock does not conform to the shape of the surface, is likely to be poorly predicted if the tangent-slab model is used.

Acknowledgment

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Finite Element Simulation of Radiative Heat Transfer in Absorbing and Scattering Media

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Introduction

NUMERICAL solutions of the radiative-transfer equation in an absorbing, emitting, and scattering medium require considerable effort in most practical systems filled with semitransparent media. Recently, many numerical methods have been developed to solve the problem of radiative heat transfer in semitransparent media, for example, Monte Carlo method, zonal method, discrete ordinates method, spherical harmonics method, diffusion approximation, finite element method (FEM), and so on.

FEM has the following advantages: 1) the approximation for the field variables in a volume or surface element can vary across the element, and 2) the variation of field variables in the FEM can be specified to increase degrees of approximation. Because of its advantage, FEM was used by many researchers to solve the problems of radiative heat transfer in semitransparent media. Razzaque et al.[1] studied the finite element solution of radiative heat transfer in a two-dimensional rectangular enclosure with gray participating media. Lin[2] developed formal integral equations describing radiative transfer in an arbitrary isotropically scattering medium enclosed by diffuse surfaces. Anteby et al.[3] used FEM to calculate the combined conduction and radiation transient heat transfer in a semitransparent medium. Furmanski and Bannaszek[4] applied FEM to solve the coupled conduction and radiation heat transfer in participating media. The conventional finite element simulations for radiative heat transfer are all based on the concept similar to zone method, in which the incoming intensity or source function is formulated as a formal integration by relating it to all surface and volume elements. Therefore, very complex geometrical integration was needed, especially for the problem of multidimensional irregular geometry.

Based on the discrete-ordinate equations of radiative transfer, Fiveland and Jessee[5] developed a finite element formulation of discrete-ordinates method for multidimensional geometries, in which the even parity radiative transfer equation is formulated for an absorbing, isotropically scattering, and reemitting medium enclosed by gray walls. The even parity radiative-transfer equation is a second-order form of transport equation, and hence the complex geometrical integration is avoided in finite element simulation. However, the even parity radiative transfer equation used by Fiveland and Jessee[5] is derived from the assumption of isotropically scattering, and cannot be used for anisotropically scattering media.

In this Note, to avoid the complex geometrical integration and consider the anisotropically scattering we develop a finite element formulation of radiative transfer based on the original discrete-ordinate equations. Two cases of radiative heat transfer in two-dimensional rectangular enclosure filled by semitransparent media are examined to verify this new formulation.

Mathematical Formulation

Consider the radiative transfer in the enclosure filled with semitransparent media. The discrete-ordinate equations of radiative transfer can be written as[6,7]

$$\mu_m \frac{\partial I^m}{\partial x} + \eta_m \frac{\partial I^m}{\partial y} + \xi_m \frac{\partial I^m}{\partial z} = -(\kappa+\sigma) I^m + \kappa I_b + \frac{\sigma}{4\pi}\sum_{m'=1}^{M} I^{m'}\Phi^{m'm}w' \tag{1}$$

with boundary conditions

$$I_w^m = \varepsilon_w I_{bw} + \frac{1-\varepsilon_w}{\pi}\sum_{|n_w\cdot s_{m'}|<0} I_w^{m'}|\boldsymbol{n}_w\cdot \boldsymbol{s}_{m'}|w_{m'} \tag{2}$$

where I^m is the radiation intensity at the direction m; I_b is the intensity of blackbody radiation at the temperature of the medium; κ and σ are the absorption and scattering coefficients of the medium, respectively; Φ is the scattering phase function; ε_w is the wall emissivity; $\boldsymbol{n}_w$ is the unit normal vector of boundary surface; $\boldsymbol{s}_{m'}$ is the unit vector in the direction m'; μ_m, η_m, and ξ_m are the direction cosine; and w_m is the weight corresponding to the direction m. By removing the forward scattering from right side of Eq. (1) to the left side, Eq. (1) can be rewritten as[8]

$$\mu_m \frac{\partial I^m}{\partial x} + \eta_m \frac{\partial I^m}{\partial y} + \xi_m \frac{\partial I^m}{\partial z} + \left(\kappa + \sigma - \frac{\sigma}{4\pi}\Phi^{mm}w\right) I^m = \kappa I_b + \frac{\sigma}{4\pi}\sum_{m'=1, m'\neq m}^{M} I^{m'}\Phi^{m'm}w', \qquad m = 1,2,\ldots,M \tag{3}$$

By using the shape function, an approximate solution of I^m is assumed in the form

$$I^m = \sum_{l=1}^{N} I_l^m \varphi_l \tag{4}$$

where the I_l^m is the values at the node l and φ_l is the shape function. The weighted residuals approach is used to spatially discretize the discrete ordinate equations [Eq. (3)]. Taking shape function φ_l as the weight function, Eq. (3) is weighted over the domain of interest

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and its integrated residuals are set to zero:

$$R_l^m = \int_V \left[\mu_m \frac{\partial I^m}{\partial x} + \eta_m \frac{\partial I^m}{\partial y} + \xi_m \frac{\partial I^m}{\partial z} + \left(\kappa + \sigma - \frac{\sigma}{4\pi}\Phi^{mm} w\right) I^m - \left(\kappa I_b + \frac{\sigma}{4\pi} \sum_{m'=1, m' \neq m}^{M} I^{m'} \Phi^{m'm} w'\right)\right] \varphi_l \, \mathrm{d}V = 0$$

$$m = 1, 2, \ldots, M, \qquad l = 1, 2, \ldots, N \quad (5)$$

The domain of interest is subdivided, and Eq. (5) is written for each element. Using isoparametric elements, the matrix system of equations can be symbolically written as

$$a_{ij}^m I_j^m = b_i^m, \qquad m = 1, 2, \ldots, M \quad (6)$$

Equation (6) is solved independently for each direction, and the boundary conditions [Eq. (2)] must be imposed on the inflow boundary. Because the in-scattering term in the discrete-ordinates equation at the direction m contains the radiative intensities of the other direction, the global iterations similar to that used in discrete-ordinates method (DOM) are necessary to include the source and boundary conditions. The detailed procedure is as follows:

Step 1: Set the initial values of intensities equal to zero.

Step 2: Solve Eq. (6) for all discrete directions.

Step 3: Calculate the modified source terms in Eqs. (3) and (5) from the intensity values of the previous iterations.

Step 4: Calculate the nondimensional net heat fluxes. Terminate the iteration process if the specified stopping criterion is satisfied. Otherwise, go back to step 2.

Because of the characteristics of FEM, for any given discrete direction the spatial differencing scheme, such as step, diamond, and exponential schemes, is not needed. Therefore, there is not a need to add artificial diffusion like in the step scheme in the context of finite difference discretization in order to achieve a stable and nonoscillating solution.

Results and Discussions

To verify the new finite element formulation just presented above for radiative heat transfer, as shown in Fig. 1, we consider the radiative heat transfer in two-dimensional rectangular gray semitransparent media enclosed by opaque boundaries with emissivity ε_w. The optical thickness based on the side length L of rectangular enclosure is $\tau_L = (\kappa + \sigma)L = 1.0$. Similarly, the optical coordinates τ_x and τ_y are defined as $\tau_x = (\kappa + \sigma)x$ and $\tau_y = (\kappa + \sigma)y$, respectively.

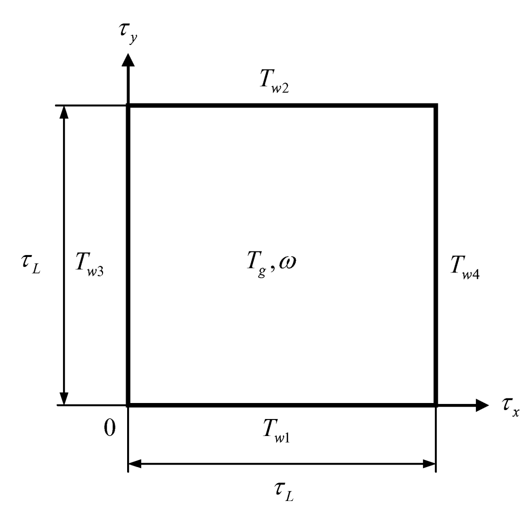

Fig. 1 Two-dimensional rectangular geometry.

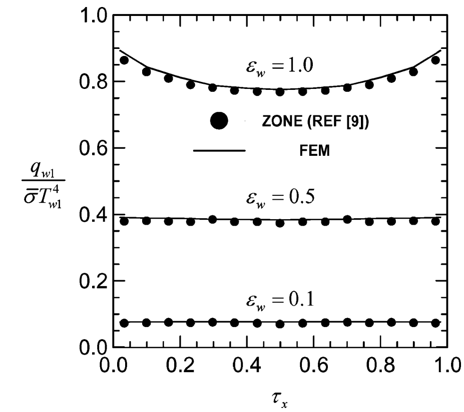

Fig. 2 Nondimensional net wall radiative heat flux in a gray enclosure with a purely scattering medium.

The temperatures of the lower, upper, left, and the right walls of the rectangular enclosure are T_{w1}, T_{w2}, T_{w3}, and T_{w4}, respectively. The temperature T_g, the absorption coefficient κ, and the scattering coefficient σ of the media enclosed by the rectangular enclosure is uniform. Two particular test cases are examined. The particular test cases are selected because exact, or at least very precise, solutions of the radiative-transfer equation exist for comparison with the finite element solution.

A computer code based on the preceding calculation procedure was written. Grid-refinement studies were also performed for the physical model to ensure that the essential physics are independent of grid size. For the following numerical study, the enclosure is subdivided into 204 triangular elements, and the equal weight even quadrature S_6 is used. The maximum relative error 10^{-4} of the nondimensional net wall heat flux is taken as the stopping criterion of iteration.

Case 1: Isotropically Scattering in a Gray Enclosure

The finite element method is applied to a rectangular enclosure filled by isotropically scattering medium with the single scattering albedo $\omega = 1.0$. The lower wall is kept hot, but all other walls and the media enclosed by the rectangular enclosure are kept cold ($T_{w2} = T_{w3} = T_{w4} = T_g = 0\text{K}$). The nondimensional net radiative heat fluxes $q_{w1}/\bar{\sigma} T_{w1}^4$ on the lower wall are presented in Fig. 2 for three values of wall emissivities, namely, 0.1, 0.5, and 1.0, and compared to the results obtained from zone method.[9] The FEM results agree with those of zone method very well. Even at blackbody wall condition ($\varepsilon_w = 1.0$), the maximum relative error is less than 3%.

Case 2: Anisotropically Scattering in a Black Enclosure

In this case, the radiative heat transfer in a square enclosure with black walls and an anisotropically scattering medium is studied. The medium is kept hot, but the temperatures of all of the boundary walls are kept as 0K. Kim and Lee[10] studied the case by DOM. The following phase function[10] with asymmetry factor 0.66972 is used:

$$\Phi = \sum_{j=0}^{8} C_j P_j(\mu) \quad (7)$$

where P_j is the Legendre polynomial. The C_j are the expansion coefficients defined as $C_0 = 1.0$, $C_1 = 2.00917$, $C_2 = 1.56339$, $C_3 = 0.67407$, $C_4 = 0.22215$, $C_5 = 0.04725$, $C_6 = 0.00671$, $C_7 = 0.00068$, and $C_8 = 0.00005$, respectively.

The nondimensional net radiative heat fluxes $q_{w1}/\bar{\sigma} T_g^4$ on the lower wall are shown in Fig. 3 for three values of single scattering albedo ω, namely, 0.0, 0.5, and 0.9, and compared to the results obtained from DOM.[10] By comparison, it can be seen that the finite element formulation presented in this Note has a good accuracy in solving the radiative heat transfer in anisotropically scattering media. Because of the characteristics of DOM, the results obtained from FEM based on the discrete ordinate equation also suffered the

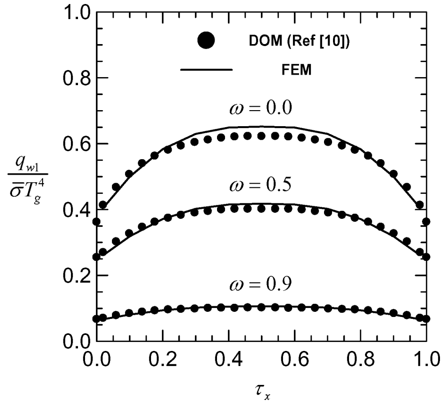

Fig. 3 Nondimensional net wall radiative heat flux in a black enclosure with an anisotropically scattering medium.

ray effect and false scattering. The discrepancies for the case of $\omega = 0.0$ are attributable to ray effect and false scattering. Even in the case of $\omega = 0.0$, the maximum relative error is less than 7%. In case 2, the number of iteration is less than six.

Conclusions

A finite element formulation based on the original discrete-ordinate equation is developed for the simulation of radiative heat transfer in absorbing and scattering media. Two cases of radiative heat transfer in two-dimensional rectangular enclosure filled with semitransparent media are examined to verify this new formulation. The results show that the finite element formulation presented in this Note has a good accuracy in solving the radiative heat transfer in absorbing and scattering media. In comparison with the conventional finite element method for radiative heat transfer, the new finite element formulation avoids the complex geometrical integration and can be used to solve the radiative heat transfer in anisotropically scattering media.

Acknowledgment

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Laminar Flow Through a Staggered Tube Bank

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Nomenclature

C_f	=	friction coefficient
C_p	=	specific heat, J/kg/K
D	=	tube diameter, m
f_c	=	average friction factor
L_{ds}	=	downstream length, m
L_{tb}	=	tube bank length, m
L_{us}	=	upstream length, m
N_L	=	number of tube rows
Nu_{LM}	=	average Nusselt number
P	=	pressure, N/m^2
Pr	=	Prandtl number ($= \mu C_p/k$)
Re_{max}	=	Reynolds number based on the maximum velocity ($= \rho V_{max} D/\mu$)
S_L	=	longitudinal pitch, m
S_T	=	transverse pitch, m
T	=	temperature, K
u_*	=	friction velocity, m/s
V_{max}	=	average velocity at minimum flow cross section
y	=	wall coordinate, m
y^+	=	wall unit or nondimensional wall coordinate ($= yu_*/\nu$)
Δy	=	distance measured from the first node to the wall, m
Δy^+	=	the first node's wall unit ($= \Delta y u_*/\nu$)
μ	=	fluid dynamic viscosity, (kg/m) s^{-1}
ρ	=	fluid density, kg/m^3

Subscripts

out	=	outlet condition
w	=	tube wall
∞	=	inlet condition

Introduction

STUDY of flow and heat transfer in tube bundles has a variety of applications in industry. Considerable effort has been spent on both experimental investigation and numerical simulation, as summarized in an earlier paper.[1] The purpose of this study is to establish that an in-house computational fluid dynamics (CFD) program is accurate by comparing results obtained with the software FLUENT with solutions obtained in the numerical study recently conducted by Wang et al.[1] The solutions for $Re_{max} = 100$ and 300 at a nominal pitch-to-diameter ratio of 1.5 are compared.

Problem Description

The flow is modeled as two-dimensional axisymmetric. The tube arrangements and the solution domain are illustrated in Figs. 1a and 2a of Ref. 1. The longitudinal pitch S_L of the tube bank is equal

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